HPC Bologna 24 June 2014

MULTIPOLAR NON-LINEARITIES

AND

CORRECTION STRATEGIES

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NORMAL FORM ANALYSIS: ARES

Tracking along a magnetic lattice (or a nonlinear optic device) reduces to the 'composition' of a sequence of symplectic maps. The one-turn map (Poicaré map) is given

$$\mathcal{M} = \exp(D_{H_n}) \circ \exp(D_{H_{n-1}}) \circ .... \circ \exp(D_{H_1})(x, p_x, y, p_y)$$

where the operator D_H is the usual Poisson bracket operator and the Lie transform $\exp(D_H)$ defines the phase flow of the Hamiltonian system H,

In the case of a linear maps (dipole and quadrupoles), the one turn map can be analytically computed: the eigenvalue at the elliptic fixed point (the origin) define the linear tunes.
In the non-linear case $\mathcal{M}$ can be computed only using a perturbative approach whose orders correspond to the non linear multiples in the magnetic lattice. But the truncated map $[\mathcal{M}]_m$ is not symplectic be used for long term tracking (dynamics aperture).

From a computational point of view we have developed an efficient code based on the relation

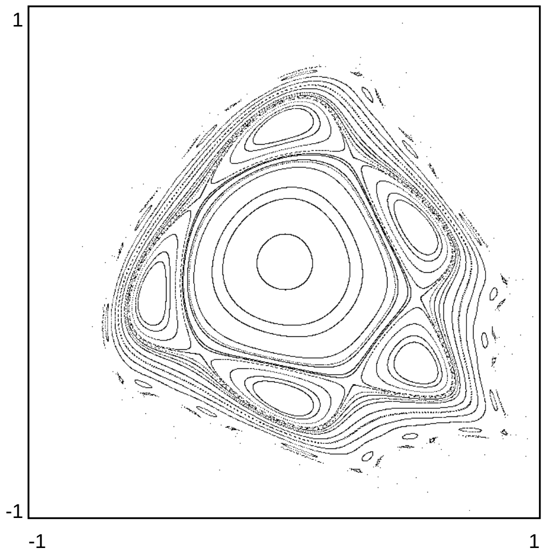
$$F\left(\exp(D_H)\mathbf{z}\right) = \exp(D_H)F(\mathbf{z})$$

to compute the polynomial composition.

The main idea of normal form is to compute a integrable sympletic map from the truncated map $[\mathcal{M}]_m$ according to the relation

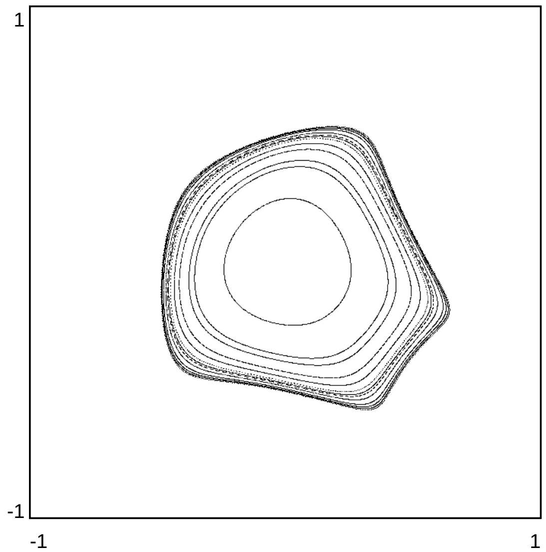
$$[\mathcal{M}]_m(\mathbf{z}) = T_m \circ \exp(D_{K_m}) \circ T_m^{-1}(\mathbf{z})$$

where $K_m(\mathbf{Z})$ is a polynomial integrable hamiltonian and $T_m(\mathbf{Z}) = \mathbf{z}$ is a polynomial change of variables.
ARES code (C language): main limitations are the exponential growth of the number of coefficients as the homogeneous polynomial order increases and the asymptotic character of the perturbative expansion that limits the applicability region (as the order increases the region shrinks to the origin).
(A. Bazzani, M. Giovannozzi, E. Todesco '*A program to compute Birkhoff normal forms of symplectic maps in* R^4', Comp. Phys. Comm. **86** (1995) 199-207).



used the following 1 parameter(s):

0.21



used the following 1 parameter(s):

0.21

Multipolar correction strategy

The non resonant normal form defined the non linear betatronic tunes

$$\Omega(I) = \omega_0 + \omega_2 I + \omega_4 I^2 + ...$$

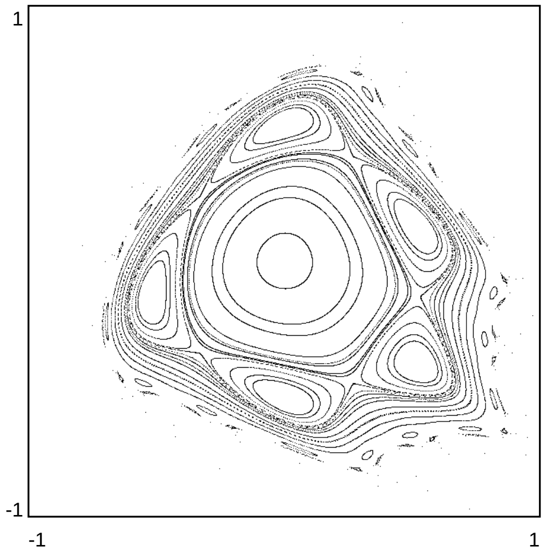
as a function of the nonlinear invariants.

$$I = \frac{P^2 + X^2}{2} + O(3)$$

Then using a family of sextupoles and octupoles in the magnetic lattice it is possible to change the one turn map in order to set $\omega_2 = 0$ to reduce the nonlinear tune-shift (W. Scandale, F. Schmidt, E. Todesco '*Compensation of the tune shift in the LHC, using the normal forms techniques*', Part. Accel. **35** (1991) 53-81). But this does not means automatically an increase of the dynamic aperture.

Resonant normal forms and dynamics aperture

It is possible to preserve the integrable character of the normal form dynamics by considering a single non linear resonance in the phase space.



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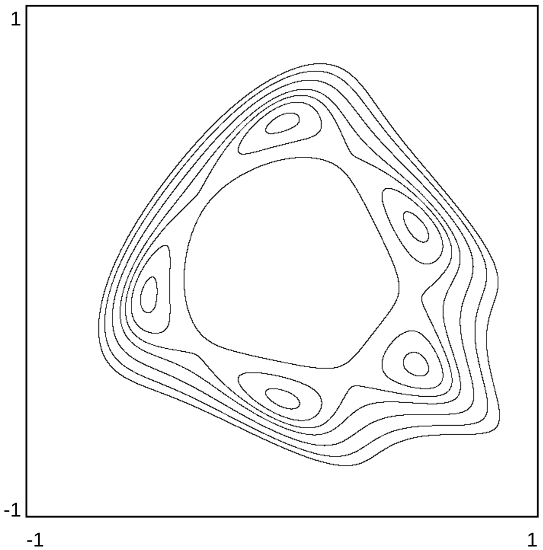


used the following 1 parameter(s):

0.21

We have developed the code NERO that evaluates the stability, the position and the width of resonances in four-dimensional symplectic maps. (E. Todesco, M. Gemmi, M. Giovannozzi. "*NERO: a code for the nonlinear evaluation of resonances in one-turn mappings*", Comp. Phys. Commun. **106** (1997) 169-80).

The resonant normal forms introduce the concept of interpolating Hamiltonian (A. Bazzani, E. Todesco, G. Turchetti, G. Servizi, *A normal form approach to the theory of nonlinear betatronic motion*', CERN Yellow Report **94-02** (1994) 225). which is relevant to study the diffusion in the phase space when a random noise or a slow parametric modulation is present (M. Martín, A. Bazzani, P.M. Cincotta, M. Pablo and C.M. Giordano, *Stochastic approach to diffusion inside the chaotic layer of a resonance* Phys. Rev E. **89-1**, 012911, (2014)).

Nekhoroshev's estimate for the dynamic aperture

A theoretical result on the normal form error suggests that the orbit transport in the phase space is bounded by a stability time

$$T_s(r) \propto \exp\left(a\left(\frac{r_*}{r}\right)^{\eta}\right)$$

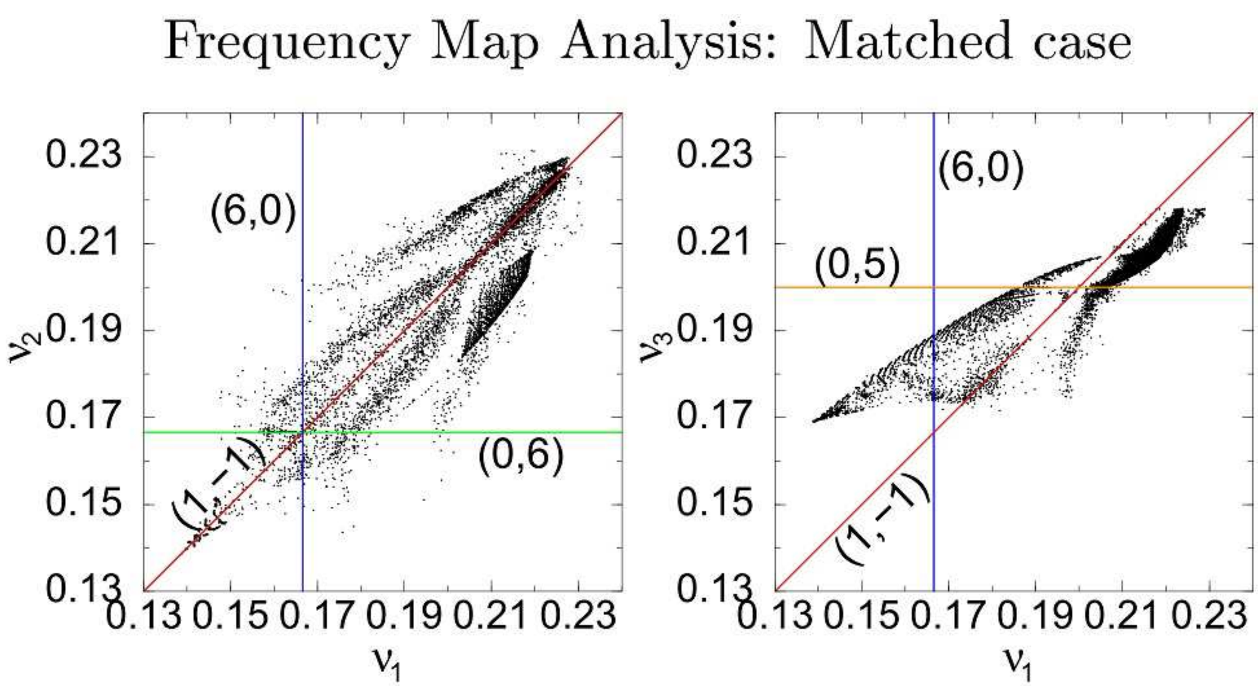
But tracking simulation suggests

$$T_s(r) \propto \exp\left(a\left(\frac{r_*}{r - r_\infty}\right)^{\eta}\right)$$

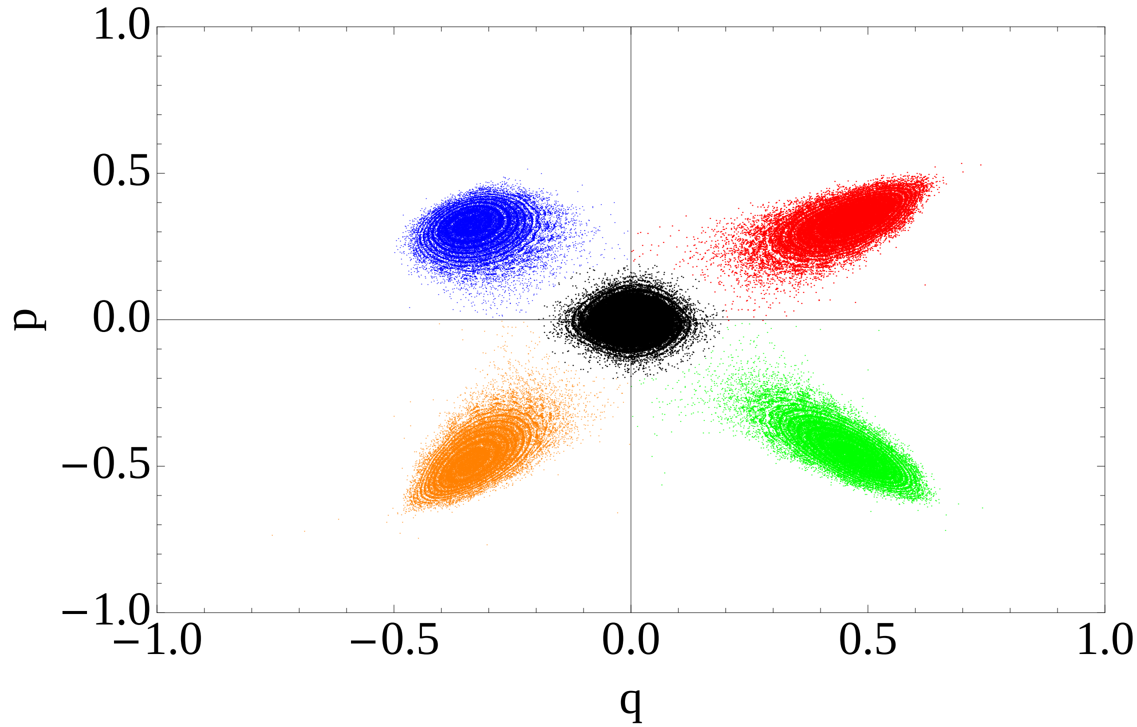
which introduces a long term dynamics aperture r_∞ which can be numerically computed using tracking simulations (M. Giovannozzi, W. Scandale, E. Todesco '*Inverse logarithmic extrapolation of survival plots in hadron colliders*', ICFA Beam dynamics newsletter **12** (1996) 6-8; M. Giovannozzi, W. Scandale and E. Todesco '*Dynamic aperture extrapolation in presence of tune modulation*', Phys. Rev. E**57** (1998) 3432).

Frequency Map Analysis

The tracking data for the betatronic motion allows to compute the Frequency Map proposed by J.Laskar using the quasi-periodic character of the regular orbits. We have developed the code PLATO (M. Giovannozzi, E. Todesco, A. Bazzani, R. Bartolini '*PLATO: a program library for the analysis of nonlinear betatronic motion*', Nucl. Instrum. and Meths. A **388** (1997) 1-7) for an accurate tune evaluation based on the Hanning filter.



Adiabatic transport



Scaling laws have been proposed for the adiabatic transport in the phase space due to the trapping into non-linear resonances. This mechanism has been proposed for an efficient extraction form SPS. (A. Bazzani, C. Frye, M. Giovannozzi, C. Hernalsteens, *Analysis of adiabatic trapping for quasi-integrable area-preserving maps* Phys. Rev. E **89**, 042915, (2014).